CHAPTER 8

2.1 Vertex Features

[illegible]

[illegible]

3 OUTCOMES ON NETWORKS

[illegible]

[illegible]

[illegible] $\beta < 1$, [illegible] $\mathbf{I} - \beta W$ [illegible] $(\alpha, \beta, \eta, \gamma)$ [illegible]:

Proposition 1 *If* $\beta < 1$, $\eta\beta + \gamma = 0$, $W_{ij} = (N-1)^{-1}$ *if* $i = j$ *and* $W_{ii} = 0$, $(\alpha, \beta, \eta, \gamma)$ *is not point-identified.*

[illegible] (1993) [illegible] (2009). [illegible] (2006) [illegible] (1993). [illegible] (1), [illegible]

$$y_i = \alpha + \beta\mathbb{E}(y_j \ w) + \eta x_i + \gamma\mathbb{E}(x_j \ w) + \ _i, \quad \mathbb{E}(\quad\quad = $$

[illegible]

x_i. A ... (1993) ... x ... (p. 535). ... P ... 1 ... x, ... $\mathbb{V}(\mathbb{E}(x_j\ w)) = 0$, ...

A ... (2) ... . I ... $\mathbb{V}(\ \mathbf{x}) = \sigma^2\mathbf{I}$, ...

$$\mathbb{V}(\mathbf{y}\ \mathbf{x}) = \sigma^2(\mathbf{I} - \beta W)^{-2}.$$

... β ... $W_{ij} = (N-1)^{-1}$ ... $i = j$ ... $W_{ii} = 0$. I ... M ... (2001) ... F ... , 1966; B ... P ... , 1986; H ... , N ... , 1987). B ... , I ... N ... .[12]

Proposition 2 *If* $\beta < 1$, $W_{ij} = (N-1)^{-1}$ *if* $i = j$, $W_{ii} = 0$, *and* $\mathbb{V}(\ \mathbf{x}) = \sigma^2\mathbf{I}$ *then* $(\alpha, \beta, \eta, \gamma)$ *is point-identified.*

I ... N ... 3. ... : I ... i' ... β ... j. I ... k ... i' ... k ... j' ... i' ... (O ... $N = 2$ ... -1. ... $N > 2$.[13]) A ... (...), P ... 2 ... .

[12] C ...

[13] ... $N = 3$, ... P ... 3 ... -0.45. F ... -0.50.

Proposition 3 *If* $\beta < 1$, $W_{ij} = (N-1)^{-1}$ *if* $i \neq j$, $W_{ii} = 0$, *and* $\mathbb{V}(\epsilon \mid \mathbf{x}) = \sigma^2 \mathbf{I}$ *then*

$$\frac{\mathbb{C}(y_i, y_j \mid \mathbf{x})}{\mathbb{V}(y_i \mid \mathbf{x})} > \frac{4-3N}{4N^2 - 11N + 8}.$$

[illegible] Proposition 3 [illegible] $l = 1, \ldots, L$, [illegible] α_l), Davezies, d'Haultfœuille, and Fougère (2009) [illegible] $\mathbb{V}(\epsilon \mid \mathbf{x}) = \sigma^2 \mathbf{I}$ [illegible] *at least* [illegible] Proposition 3.2). [illegible] (2015) [illegible] $\mathbb{V}(\epsilon \mid \mathbf{x}) = \sigma^2 \mathbf{I} + \sigma (W + W')$. [illegible] W [illegible] Bramoullé, Djebbari, and Fortin (2009) [illegible]).

[illegible] (2) [illegible] Glaeser, Sacerdote, and Scheinkman, 1996 [illegible]). Graham (2008), [illegible] $l = 1, \ldots, L$ [illegible]

$$\mathbf{y}_{l} = \gamma W_{l} \mathbf{y}_{l} + \alpha_l \mathbf{1}_{N_l} + \epsilon_{l},$$

[illegible] N_l [illegible] l, $W_{ij,l} = (N_l - 1)^{-1}$ [illegible] $i \neq j$ and $W_{ii,l} = 0$, [illegible] α_l [illegible].[14] (A [illegible] Glaeser, Sacerdote, and Scheinkman, 2003.) [illegible] $\epsilon_{i,l}$, [illegible] $\sum_{j \neq i} \epsilon_{j,l}/(N_l - 1)$, [illegible] α_l. [illegible]

[illegible] (2015) [illegible] (2) [illegible] 5).

A [illegible] (2) [illegible] B[illegible], D[illegible] [illegible] F[illegible] (2009):

Proposition 4 (B[illegible], D[illegible] [illegible] F[illegible], 2009) *If* $\eta\beta + \gamma = 0$ *and* $\mathbf{I}$, W, W^2 *are linearly independent,* $(\alpha, \beta, \eta, \gamma)$ *is point-identified.*

I[illegible] $W_{ij} = (N-1)^{-1}$ [illegible] $i = j$ [illegible] $W_{ii} = 0$, $W^2 = (N-1)^{-1}\mathbf{I} +$

F t e 1 De é d C é N .

e e . Te a W^2 t a (W^2

[illegible]

... (2015).

I ... d) ...). ... $O(N^2)$ ... $(\mathbf{y}, \mathbf{x})$ ... (e.., $TN > N^2$). ... N ... .[21] ... . M ... (2011) ... 1 ... .[22] A ... (2014) ... 3 ... .[23] I ... AddH ... 2 ... .

... (LA...O) (T... 1996; ... B... , C... , ... d H... , 2013, ...), ... C... d A... (...CAD) ... (F... d L., 2001), ... E... N ... (... d H... , 2005), ... M... C... P... (MCP) (... , 2010). I T ... $(\mathbf{y}, \mathbf{x})$

[illegible]

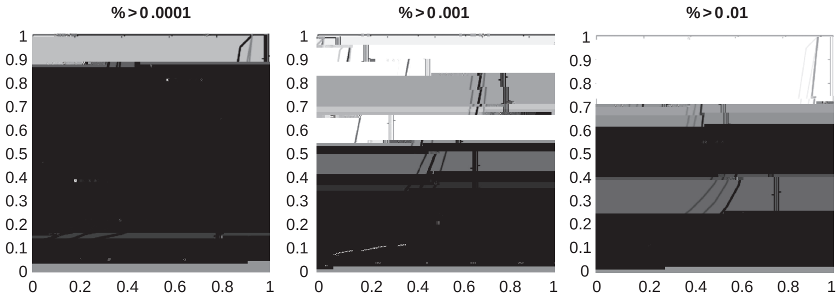

Figure 3 [illegible] De[illegible] d C[illegible]. *Note:* [illegible] $(\mathbf{I}-\beta W)^{-1}(\mathbf{I}+W)$ [illegible] 0.0001, 0.001 [illegible] 0.01, [illegible] β. [illegible] W [illegible] 100 [illegible] 100 [illegible] $(N = 100)$, [illegible] $W_{ii+1} = W_{100,1} = 1$ [illegible] $i = 1, \ldots, 100$ [illegible] W [illegible] 1 [illegible] $\mathbf{I}+W$, [illegible] $\beta = 0$, [illegible] 2 [illegible].

[illegible] A [illegible] *approximate* [illegible] (i, j) [illegible] W^k [illegible],

[illegible] W [illegible] LASSO [illegible] β.) [illegible]

[illegible] W [illegible] (2013), [illegible] W. [illegible]

$$(W,\beta,\delta,\gamma)\ \frac{1}{T}\sum_t \|\mathbf{y}_t-\alpha-\beta W\mathbf{y}_t-\eta\mathbf{x}_t-\gamma W\mathbf{x}_t\|_2^2+\lambda\sum_{i=j} p_T(W_{ij}), \tag{6}$$

[illegible] $\|\ \|_2$ [illegible] L_1 [illegible] W. [illegible] y [illegible] (2013) [illegible] it [illegible] A2). [illegible] N [illegible] T

[illegible]

[illegible]

3.2 Nonlinearities and Multiple Equilibria

[illegible]

d b t e e a t e **y**$(\mathbf{x})$, **x** 0, 1 ^{N}

[illegible] 2010, [illegible]).[31] [illegible]

4 NETWORK FORMATION

[illegible]

4.1 Statistical Models

[illegible] $(\mathcal{G}, \sigma(\mathcal{G}),$ [illegible] $\sigma(\mathcal{G})$ [illegible] $\mathcal{G}$, [illegible] $(\mathcal{G}, \sigma(\mathcal{G}))$. [illegible] N [illegible] p [illegible] $(0, 1)$ [illegible] $2^{N(N-1)/2}$ [illegible] N [illegible] (*but not necessarily more than one*). [illegible] N [illegible] $N(N-1)/2$ [illegible] .)

[illegible] (2006), [illegible] N [illegible] Np, [illegible] k [illegible] N_k

a P d b t v a a e e pN

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

A PROOFS

A.1 Proof of Proposition 2

I [illegible] $\mathbb{V}(\varepsilon|\mathbf{x}) = \sigma^2\mathbf{I}$, [illegible] $\mathbb{V}(\mathbf{y}|\mathbf{x}) = \sigma^2(\mathbf{I} - \beta W)^{-2}$. [illegible] $\beta < 1$ [illegible] W [illegible]

$$(\mathbf{I} - \beta W)^{-1} = \mathbf{I} + \beta W + \beta^2 W^2 + \dots$$

I [illegible] $W^k, k = 1, 2, \dots$ [illegible] $(W^k)_{ii} = a_{k-1}$ [illegible] $(W^k)_{ij} = a_k, i \neq j$, [illegible]

$$a_0 = 0, \qquad a_{-1} = 1 \quad \text{[illegible]} \quad a_k = (a_{k-2} + a_{k-1}(N-2))/(N-1).$$

[illegible] $S = \sum_{k=1} \beta$

$\mathbb{V}(\mathbf{y}\ \mathbf{x}) = \sigma^2(\mathbf{I} + S)^2,$

[illegible] $p(b;\overline{\kappa}, N)$ [illegible]

$$\overline{\kappa} > \frac{-(N-2)}{(N-2)^2 +}$$

[illegible]

$$\psi(\beta) = \frac{\mathbb{C}(y_{i,1}, y_{j,1}\ \mathbf{x}_1)}{\mathbb{C}(y_{i,2}, y_{j,2}\ \mathbf{x}_1)} = \frac{8 + 8\beta + 2\beta^2}{4 + 7\beta + 2\beta^2 - \beta^3}.$$

I [illegible] $\psi(\beta) < 0$ [illegible] $\beta < 1$ [illegible] $\psi(\beta) > \psi(1) = 1.5$.

[illegible] $\overline{\psi}$ [illegible] β [illegible] $q(b; \overline{\psi}) = 0$ [illegible] $q(b; \overline{\psi})$ [illegible] $-\overline{\psi}\beta^3 + 2(\overline{\psi} - 1)\beta^2 + (7\overline{\psi} - 8)\beta + 4(\overline{\psi} - 2)$. [illegible] 1 [illegible] 1987, [illegible] 225 [illegible] 229). [illegible] f [illegible] g [illegible] C [illegible] $g(x) < f(x)$ [illegible] C, [illegible] f [illegible] $f + g$ [illegible] C, [illegible] $f(x) = a_1 x$ ([illegible] $f(x) = 0$ [illegible] $x = 0$) [illegible] $g(x) = a_0 + a_2 x^2 +$ [illegible] $+ a_K x^K$ [illegible]

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